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PAPR and ICI Reduction of OFDM Signals

M Maksud Alam and Besma Smida

***Abstract*—In this work, we propose an OFDM code that guarantee both low peak to average power ratio (PAPR) and low inter-carrier interference. This code is composed of cosets of first order binary Reed-Muller codes. Our contributions are two-fold: 1) we introduce an encoding scheme that guarantee a (PAPR) by 3 dB and a significant ICI reduction, 2) we propose moderate complexity decoding procedures for this scheme. We evaluate the performance of the proposed schemes in additive white Gaussian channel. Compared to the code proposed by [1], our code performs better in term of probability of error and have significantly lower PAPRs.**

I. INTRODUCTION

A. Motivation

Orthogonal frequency-division multiplexing (OFDM) has been considered for many high-speed wireless applications. Indeed, the orthogonal properties among subcarriers of OFDM lead to high spectral efficiency and excellent ability to cope with multipath fading environment [2]. However OFDM systems have two major drawbacks: high peak-to-average ratio (PAPR) due to summing up of large number of subcarriers data together and inter-carrier interference (ICI) because the orthogonal properties are easily broken down by frequency offset errors caused by oscillator inaccuracies. In this paper, a coding technique is proposed to reduce both drawbacks.

B. Related Works

CFO Effect Reduction: To mitigate the frequency-offset problem, two types of approaches have been proposed in the literature. One is to estimate and remove the frequency offset [3]. In this approach, the frequency-offset estimation is generally performed in two steps: *Coarse frequency-offset estimation*, which estimates the part of frequency offset that is multiple of the subcarrier spacing and *fine frequency-offset estimation*, which estimates the remaining part of the offset. Another approach is to use signal processing or coding to reduce the sensitivity of the OFDM system to the frequency offset [4]. A simple and effective method known as the ICI self-cancelation scheme, proposed by Zhao and Haggman in [5] and later extended in [6], [7], [8], where polynomial coding in the frequency domain is used to mitigate the effect of frequency offset. In this method, copies of the same data symbol are modulated on r adjacent sub-carriers using optimized weights. This method can significantly reduce the ICI at the price of lowering the transmission rate by a factor r and a slight increase in complexity. The authors of [9] show the capacity of any forward-error correction code to eliminate the errors caused by the ICI. Previous attempts to decrease ICI also include correlative-coding methods proposed in [10]. More recently, Smida proposed in [11] to choose a well understood code and to rotate each coordinate of the code by a fixed phase shift to reduce ICI.

PAPR Reduction: High fluctuations in the signal amplitude of OFDM systems, generated by the constructive addition of a large number of sub-carriers, is considered as an important drawback. To alleviate this problem, known also as peak-to-mean envelope power (PMEPR) several reduction schemes have been proposed for multicarrier systems. Selected mapping (SLM) and partial transmit sequences (PTS) approaches reduce the probability of generating a signal with large PMEPR, by generating multiple statistically independent OFDM symbols for a given data frame and transmitting the symbol with the lowest peak power [13], [14], [15], [16]. Clipping and filtering techniques cut-off the peaks then to reduce the out of band radiation, generated by clipping, filter the signal [12]. Another approach is to use coding to reduce PMEPR. The theoretical aspects of the relation between the code rate, minimum Euclidean distance of the code and its block length is provided in [17] as two fundamental theorems. The first one proves a lower bound for PAPR based on the three aforementioned parameters.

While most research works study the PAPR reduction and ICI reduction separately, one important exceptions exists. It has been shown in [1] that both PAPR and ICI can be reduced by using Golay complementary repetition code.

C. Our Contributions

Motivated by the success of the previous approaches, this paper considers controlling PAPR and ICI of OFDM signals by using coding. Our code is composed of cosets of first order binary Reed-Muller codes. This approach enjoys the twin benefits of PAPR and interference reduction and is easy to implement in practice. Our code guarantees a peak to average power ratio (PAPR) by 3 dB and outperforms the code proposed by [1] in term of probability of bit error.

II. STATEMENT OF THE PROBLEM

A. Preliminaries

An OFDM signal is the sum of many independent signals modulated onto subchannels of equal bandwidth. The OFDM transmitter applies the inverse discrete Fourier transform (IDFT) operation and sends the symbol sequence to the RF chain. The transmitted signal at time t may be represented as the real part of the complex envelope

$$S_{\mathbf{c}}(t) = \sum_{n=0}^{N-1} c_n \exp(j2\pi(f_0 + nf_s)t) \quad \text{for} \quad 0 \le t \le T_s, \tag{1}$$

M Maksud Alam (Email: malam@purduecal.edu) and Besma Smida (Email:bsmida@purdue.edu) are with the Department of Electrical and Computer Engineering, Purdue University Calumet, Hammond.

where $j = \sqrt{-1}$, f_0 is the beginning frequency and f_s is the bandwidth of each subcarrier. The OFDM-symbol duration T_s is equal to $\frac{1}{f_s}$ to ensure the orthogonality among the subcarriers. The vector $\mathbf{c} = [c_0, c_1, \ldots, c_{N-1}]$ of length N is the modulating data sequence. We refer to $\mathbf{c}$ as codeword of code $\mathcal{C}$ that maps blocks of k input bits into blocks of N symbols. The data-coefficients c_n are typically taken from a constellation $\mathcal{Q}$ with 2^h elements. The *peak-to-mean envelope power ratio* (PMEPR) of the transmitted signal is

$$\mathrm{PMEPR}(\mathbf{c}) = \max_{0 \le t \le 1} \frac{|S_{\mathbf{c}}(t)|^2}{\parallel \mathbf{c} \parallel^2}. \tag{2}$$

We assume that $S_{\mathbf{c}}(t)$ is transmitted on an additive white Gaussian noise (AWGN) channel. The receiver receives the signal $\Re(S_{\mathbf{c}}(t))$ perturbed by noise and performs the inverse operations: the RF chain at the receiver down-converts, processes the received data. The receiver then applies a discrete Fourier transform (DFT) and generate estimates of $\mathbf{c}$. The receiver then extracts the block of input bits by applying a suitable error-correction algorithm. The received signal sample for the kth subcarrier after DFT can be written as

$$y_k = c_k b_0 + \sum_{l=0, l \neq k}^{N-1} b_{l-k} c_l + n_k = c_k b_0 + I_k + n_k, \tag{3}$$

for $k = 0, 1, \ldots, N-1$, where n_k is a complex Gaussian noise sample. The first term in the right-hand side of (3) represents the desired signal. The ICI term, attributable to the CFO, on the kth subcarrier is expressed by I_k. The ICI coefficients b_k is given by

$$b_k = \frac{\sin \pi(k+\varepsilon)}{N \sin \frac{\pi}{N}(k+\varepsilon)} \exp\left[j\pi(1 - \frac{1}{N})(k+\varepsilon) \right], \tag{4}$$

where ε is the normalized frequency offset defined as a ratio between the frequency offset (which remains constant over each symbol period) and the subcarrier spacing. Note that I_k is a function of both $\mathbf{c}$ and ε.

Sathanannthan and Tellambura [1] defined the Peak Interference to Carrier Ratio PICR as

$$\mathrm{PICR}(\mathbf{c}, \varepsilon) = \max_{0 \le k \le N-1} \frac{|I_k|^2}{|b_0 c_k|^2}. \tag{5}$$

PICR is maximum interference-to-signal ratio for any subcarrier. It specifies the worst-case ICI on any subcarrier. Large I_k values cause high bit errors in subcarriers. To reduce ICI effects, PICR should be minimized. Note that the expression of PICR is similar to the PMEPR, subsequently one may apply PMEPR reduction techniques to reduce PICR. To improve the performance of OFDM systems both PMEPR in (2) and PICR in (5) should be minimized.

B. Golay Complementary Sequences and Reed-Muller Codes

Our code is composed of cosets of first order binary Reed-Muller codes, specifically cosets of first order Reed-Muller codes in the second order Reed-Muller codes. This approach is based on the Golay complementary sequences. The following theorems justify this choice.

Theorem 1: For unit power constellations, the PAPR of any complementary Golay sequence is bounded by 2.

Theorem 2: For any permutation π of the set $\{1, 2, \ldots, m\}$, and for any $c_k, c \in \mathbb{Z}_{2^h}$. The sequence generated by

$$u(\mathbf{x}) = 2^{h-1} \sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)} + \sum_{k=1}^{m} c_k x_{\pi(k)} + c,$$

is Golay complementary over $\mathbb{Z}_{2^h}$ of length 2^m.

The Theorem 1, proved in [18], motivates the utilization of Golay sequences for OFDM systems. The Theorem 2 describes the link between the Golay sequences and RM codes [19]. The RM code over $\mathbb{Z}_{2^h}$ of order r and length 2^m, denoted by $\mathrm{RM}_{2^h}(r, m)$, consists of all linear combinations of vectors associated with the monomials of degree at most r in m variables. The Boolean monomial of m variables is defined as $f_x = x_0^{r_0} x_1^{r_1} \ldots x_{m-1}^{r_{m-1}}$ where $(x_0, x_1, \ldots, x_{m-1})$ is an m-tuple binary vector. Henceforth, each of $\frac{m!}{2}$ cosets of $\mathrm{RM}_{2^h}(1, m)$ in $\mathrm{RM}_{2^h}(2, m)$ having a coset representative of the form $\sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)}$ comprises $2^{h(m+1)}$ Golay sequences over $\mathbb{Z}_{2^h}$ of length 2^m. Thus, we conclude that for Reed-Muller codes over $\mathbb{Z}_{2^h}$, the PAPR of the codewords in the cosets does not exceed 3 dB [19].

III. PROPOSED TECHNIQUE: SELECTED COSET (SLC)

It has been shown in [1] that both PAPR and ICI can be reduced by using Golay complementary repetition code. The code proposed in [1] is a simple combination of Golay complementary sequences (to reduce PAPR) and repetition code (to reduce PICR). We propose in this letter a more efficient code to reduce PAPR and ICI. The main idea is to choose a subset $\mathcal{U}$ of coset representatives having the form $\sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)}$ to guarantee that the PAPR does not exceed 3 dB. The subset $\mathcal{U}$ is also used to reduce the PICR of the transmitted sequences. We take Reed-Muller codes over $\mathbb{Z}_{2^h}$ and add coset representatives $u \in \mathcal{U}$ such that the maximum inter-carrier interference (ICI) taken over all subcarriers is minimized. The size of the subset $\mathcal{U}$, $|\mathcal{U}|$, varies form 1 to $\frac{m!}{2}$. The proposed code offers the flexibility to choose the amount of PICR reduction by varying $|\mathcal{U}|$. Indeed, if we assume $|\mathcal{U}| = 1$, then we shift all codewords by a unique coset representative, this is similar to the idea proposed in [11]. If we assume $|\mathcal{U}| > 1$, then for each codeword we generate $|\mathcal{U}|$ OFDM sequences and transmit the $\widehat{u}$-th sequence with the lowest PICR as follow:

$$\widehat{\mathbf{u}} = \arg \min_{\mathbf{u} \in \mathcal{U}} \mathrm{PICR}(\mathbf{c} + \mathbf{u}, \varepsilon). \tag{6}$$

The value of $\widehat{u}$ is required at the receiver to recover the signal successfully and can be derived without side information as in [20]. Our approach enjoys the twin benefits of interference reduction and error correction. Note that the code proposed in [1] used all the $\frac{m!}{2}$ coset representatives to transmit more information. The encoding and decoding procedures are given below.

Encoding

1) Use $\mathrm{RM}(1, m)$ to encode each $m+1$ bits into codeword of length 2^m.

2) Find the coset representative producing the lowest PICR as in (6).
3) Shift the codewords by the selective coset leader.
4) Map each $\mathbb{Z}_2$ symbol to the BPSK or QPSK constellation.

Decoding

1) Extract each soft bit from BPSK or QPSK symbol.
2) Compensate for all $|\mathcal{U}|$ possible coset leaders.
3) Find the best match, i.e. the coset leader that maximize the average Log-Likehood Ratio (LLR).
4) Shift the codeword by the coset leader.
5) Perform regular RM decoder.

IV. SIMULATION RESULTS

The simulation results were obtained for an OFDM system with $N = 16$, so we can compare with the Golay complementary repetition codes [1]. We also assume BPSK modulation and AWGN channel with various normalized frequency offsets ε. We used the following coset representative $[0001010000011011]$ ($|\mathcal{U}| = 1$). The PAPR of the proposed SLC is limited to 3 dB regardless of the number of subcarriers. Our scheme has a power back-off gain of 3 dB over Golay complementary repetition codes. Fig. 1 shows the complementary cumulative distribution function (CCDF) of PICR. The SLC codes show PICR reduction of 2.5 dB. Fig. 2 provides the BER performance of SLC and Golay complementary repetition codes of exactly the same rate $R = \frac{5}{16}$. Note that SLC outperform the Golay complementary repetition technique for all ε. For a probability of 10^{-5} the gains are 2.1 dB, 1.9 dB, and 0.2 dB for $\varepsilon = 0.1$, $\varepsilon = 0.2$, and $\varepsilon = 0.3$, respectively. The BER improvement of SLC is due to the combined effect of ICI reduction (choice of coset representative) and the BER performance of the Reed-Muller code.

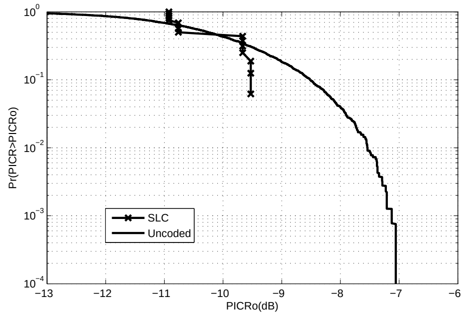

Fig. 1. PICR of SLC and uncoded OFDM with normalized frequency offset, $\varepsilon = 0.1$.

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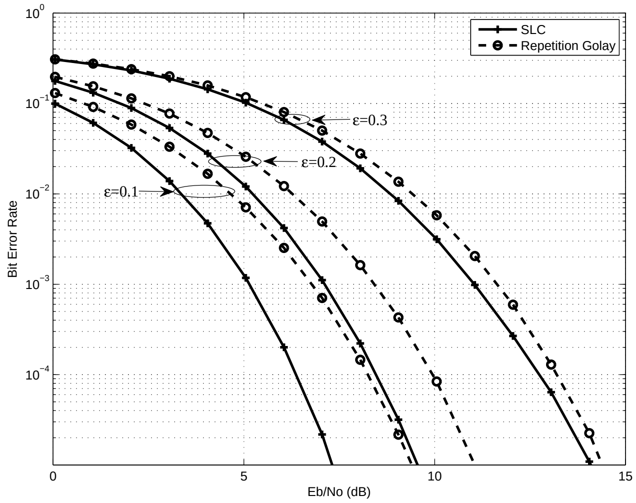

Fig. 2. Comparison of SLC and Golay complementary repetition codes with various normalized frequency offsets.

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